Chapter 12
Physical Laws and Worldlines in Minkowski Spacetime

Vesselin Petkov

Abstract In his paper “Space and Time” a hundred years ago Minkowski gave us the adequate relativistic picture of the world. According to him what exists is an absolute four-dimensional world in which the ordinary physical bodies are worldlines. Minkowski conjectured that physical laws might find their most perfect expression as interrelations between these worldlines. The purpose of this paper is to examine further whether Minkowski’s idea can be applied to different areas of physics. It is shown that not only does it work perfectly in classical physics and general relativity, but also provides a deeper understanding of some difficult questions (including the origin of inertia) and demonstrates that taking seriously the existence of worldlines inescapably leads to the concept of gravity as curvature of spacetime. It is also shown that expanding Minkowski’s idea to quantum physics might shed light even on the nature of the quantum object.

Keywords Minkowski spacetime · Worldlines · Worldtubes · Physical laws · Origin of inertia · Four-dimensional stress · Gravity · Quantum object · Discontinuous existence in time

12.1 Introduction

On September 21, 1908 in his talk “Space and Time” Hermann Minkowski proposed a radical change of our views of space and time – “space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality” [1]. Minkowski arrived at the view that space and time form an independent four-dimensional reality, which he called “the world,” by analyzing and successfully revealing the profound meaning of the

V. Petkov (✉)
Science College, Concordia University, Montreal, QC H3G 1M8, Canada
e-mail: vpetkov@alcor.concordia.ca

V. Petkov (ed.), *Minkowski Spacetime: A Hundred Years Later*,
Fundamental Theories of Physics 165, DOI 10.1007/978-90-481-3475-5_12,

relativity postulate – “the postulate comes to mean that only the four-dimensional world in space and time is given by the phenomena” [1]. He wrote: “the word *relativity-postulate* [...] seems to me very feeble” [1] and preferred “to call it the *postulate of the absolute world*” [1]. This insight also appears to have been helped, as we will see in Section 12.2, by his realization that the consequences of the special theory of relativity are manifestations of the four-dimensionality of the world. In this absolute world, which turned out to be totally counter-intuitive, space and time have equal status – all points of space and all moments of time have equal existence since they form the dimensions of the absolute four-dimensional world of Minkowski, which we now call Minkowski spacetime or simply spacetime.

As the myriad of macroscopic physical bodies which move in the ordinary three-dimensional world are represented by a forever given network of timelike worldlines in spacetime, Minkowski conjectured that there is a close link between physical laws and the network of worldlines: “The whole world seems to resolve itself into such world-lines, and I would fain anticipate myself by saying that in my opinion the laws of physics might find their most perfect expression as interrelations between these world-lines” [1]. This appears to be a natural conjecture since Minkowski seems to have believed[1] that spacetime is not merely an abstract four-dimensional mathematical space, but represents a real four-dimensional world of one temporal and three spatial dimensions. In this world the perceived macroscopic three-dimensional bodies are *real* four-dimensional worldlines or rather worldtubes since we are dealing with spatially extended physical bodies. As physical laws govern the interactions of physical bodies in the three-dimensional world of our perceptions it does appear to follow that physical laws should be directly linked to the network of the bodies’ worldtubes in spacetime since only these worldtubes exist there; there are no three-dimensional bodies and no dynamic interactions in the static Minkowski world in the ordinary sense since all interactions are fully realized there.

To my knowledge no consistent attempt has been made so far to examine Minkowski’s conjecture. The purpose of this paper is precisely this – to analyze a number of physical laws in order to determine whether or not they are expressions of the interrelations between the worldtubes of macroscopic physical bodies that participate in interactions governed by those physical laws. The analysis will be extended beyond Minkowski’s original idea and will try to determine whether quantum laws can be expressed in terms of spacetime structures, not worldlines since quantum objects cannot be represented by worldlines in spacetime. To perform such

---

[1] Not everyone agrees that Minkowski considered reality to be a four-dimensional world. Although I think he did regard the world as four-dimensional since he appears to have realized that the relativistic effects are manifestations of its four-dimensionality (as will be demonstrated in Section 12.2), it is obvious that what he thought about spacetime does not determine its status of existence.

analyses it is first necessary to address the question of the reality of the macroscopic bodies' worldtubes and ultimately the reality[2] of Minkowski spacetime itself.

Section 12.2 deals with the most important issue raised by Minkowski's paper "Space and Time" – whether the real world is four-dimensional with time being the fourth dimension (which would mean that physical bodies are real timelike four-dimensional worldtubes) and whether the relativistic effects are indeed manifestations of the four-dimensionality of the world as Minkowski anticipated. Section 12.3 demonstrates that the mechanical pre-relativistic and pre-quantum physical laws – Newton's three laws – can be naturally explained as interrelations between worldlines. Section 12.4 explores the internal logic of the concept of gravity as a force (an analysis that appears to have been mostly followed by Einstein) which inescapably leads to the concept of gravity as curvature of spacetime according to which gravitational interaction is expressed as interrelations between geodesic worldlines in curved spacetime. Finally, Section 12.5 examines whether Minkowski's program applies to quantum physics where the quantum objects are not represented by worldlines.

12.2 Minkowski Spacetime and Reality

The major implication of Minkowski's paper "Space and Time" is the issue of the reality of Minkowski spacetime[3]: Is it just a four-dimensional mathematical space or is it representing a real four-dimensional world with one temporal and three spatial dimensions?

To address this issue one should start with an explicit definition of the macroscopic[4] reality (everything that exists at the macro scale). One feature of the world that appears to be self-evident is that it is three-dimensional[5] – macroscopic physical bodies are three-dimensional and space itself is three-dimensional (one can talk

---

[2] In this paper by "reality of spacetime" I mean the reality of the four-dimensional world as envisioned by Minkowski in which time is entirely given as the fourth dimension and macroscopic physical bodies are four-dimensional worldtubes. The issue of reality of spacetime in terms of the debate absolutism (substantivalism) versus relationism will be mentioned only once in Section 12.4 and will not be discussed here.

[3] It does not matter whether we talk about Minkowski spacetime or any other relativistic spacetime if we ask the question of their reality. What is common to all spacetimes is their four-dimensionality. So by asking whether spacetime is real we ask whether the world is four-dimensional. A real four-dimensional world could be flat (represented by Minkowski spacetime) or curved (represented by another relativistic spacetime).

[4] Relativity describes mostly the macro scale of the world and does not fully apply at the quantum level since its equations of motion manifestly fail to describe the behaviour of quantum objects. The equations of motion of relativity (special or general, depending on the case) govern the behaviour of the perceived macroscopic three-dimensional bodies. That is why it is the macroscopic three-dimensional bodies (not quantum objects) that are represented by timelike worldtubes in relativity.

[5] Even Aristotle regarded reality as a three-dimensional world [2, p. 359].

about distances between bodies in terms of projections only along three mutually orthogonal directions). Another feature which also appears to be self-evident is that everything that exists, exists only at the present moment which constantly changes. The next question is how what we perceive as a three-dimensional space and three-dimensional bodies exist at the present moment. The first, naive view of reality could be defined in the following way – what exists is everything that we *see* (or can in principle see) *simultaneously* at the moment "now"; we "see" space through the distances between objects. So, the four key defining features of the first naive view of reality can be summarized in four words – three-dimensional, see, simultaneously, and "now".

However, when Rømer determined that the speed of light was finite in 1676 it became clear that what we see simultaneously at the moment 'now' is all past since light needs some time to reach our eyes. It turned out that the space we believe we "see" at the present moment does not constitute even a space since it consists of space volumes (around and between bodies) which correspond to different past moments, whereas space is defined in terms of *simultaneity* – as all space points at a given moment of time.

The view of reality established after Rømer's discovery recognized as real everything that *exists simultaneously* at the moment "now". The defining features of this view had not changed significantly and could be summarized again in four words – three-dimensional, exist, simultaneously, and "now". One of these features – the reference to a single moment of time (the present moment) – is an assumption that is based *solely* on the fact that we are aware of ourselves and the world only at that moment. However, it is evident that it does not necessarily follow from this fact that the whole world also exists only at the present moment.[6]

On this view of reality, called *presentism*, all that exists is the present. The present itself is defined as the three-dimensional world – as everything that exists *simultaneously* at the present moment. This pre-relativistic view is still widely accepted not only by the general public but also by some scientists and philosophers. This is a disturbing fact since presentism clearly contradicts the theory of relativity. As the present is defined in terms of *absolute simultaneity* – as everything that exists simultaneously at the moment "now" – if it were only the present that existed, it follows that all observers in relative motion would share the same present (the same set of simultaneous events) which would mean that simultaneity is absolute in contradiction with special relativity.

Then the natural question is: "What is reality according to special relativity?" Or more precisely: "What is the dimensionality of the world at the macroscopic scale where special relativity is fully[7] applicable?" Apparently Minkowski had asked him-

---

[6] We are aware of ourselves at a specific location in space, but we do not assume that everything that exists, also exists solely at that location.

[7] As indicated in footnote 4 special relativity is a macroscopic theory in its *entirety* and does not fully apply at the quantum level.

self such questions[8] and arrived at the relativistic view of reality – what exists is an absolute four-dimensional world consisting of one temporal and three spatial dimensions. As mentioned in the Introduction he did that by analyzing the physical meaning of the relativity principle – "physical laws would be expressed in exactly the same way by means of x', y, z, t' as by means of x, y, z, t" [1], where x', y, z, t' and x, y, z, t represent the systems of reference of two observers moving uniformly with respect to each other along their x-axes. Minkowski seems to have recognized the far reaching implications of one specific fact following from Einstein's formulation of special relativity – that the times t and t' should be treated equally. The first immediate implication[9] is that *different* times mean *different* classes of simultaneous events (relativity of simultaneity). And as space is defined in terms of *simultaneity* – the class of simultaneous events corresponding to a given moment of time – it follows that the primed observer should "define space by the manifold of the three parameters x', y, z" [1]. Therefore, the two observers in relative motion have not only different times but also *different* three-dimensional spaces.[10] That is why Minkowski's conclusion was unavoidable: "We would then have in the world no longer *the* space, but an infinite number of spaces, analogously as there are in three-dimensional space an infinite number of planes. Three-dimensional geometry becomes a chapter in four-dimensional physics. You see why I said at the outset that space and time are to fade away into shadows, and that only a world in itself will subsist." [1].

---

[8] In my view a clear indication of this is the fact that he discussed the description of what exists – he used the word "the *world*" (in which we can "imagine that everywhere and everywhen there is something perceptible" [1]; that "only a world in itself will subsist" [1]), the word "universe" (which is filled with worldlines of "something perceptible" [1]), "our concept of nature" [1], "four-dimensional physics" [1], and physical objects which are represented by four-dimensional bands [1].

[9] As, on the pre-relativist view, a class of simultaneous events corresponds to each moment of the absolute time, it does follow that two times entail different classes of simultaneous events corresponding to each moment of the two times. In other words, at each moment of their times two observers in relative motion will have different classes of simultaneous events.

[10] Minkowski noticed that "neither Einstein nor Lorentz made any attack on the concept of space" [1]. This is an undeniable triumph of a great mathematician over two great physicists. To realize that different times imply different spaces as well, could have been perhaps also realized by Einstein especially given the fact that Minkowski realized it almost immediately after Einstein insisted that the times t and t' of two observers in relative motion should be treated on equal footing. The omission to notice that different times imply different spaces is more easily explainable in the case of Lorentz. He did not regard the times t and t' of two observers in relative motion as equal since he did not believe t' represented anything real. This meant that an introduction of another space for the primed observer would not be justified. In 1915, in a note added in the second edition of his book "The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat" [8] Lorentz himself described the failure of his attempts to formulate properly the theory of relativity: "The chief cause of my failure was my clinging to the idea that the variable t only can be considered as the true time and that my local time t' must be regarded as no more than an auxiliary mathematical quantity. In Einstein's theory, on the contrary, t' plays the same part as t; if we want to describe phenomena in terms of x', y', z', t' we must work with these variables exactly as we could do with x, y, z, t."

Now the profound meaning of the relativity postulate as revealed by Minkowski becomes completely evident – physical laws are the same in all inertial reference frames in relative motion because each inertial frame has its own space and time and each inertial observer describes the physical laws in his or her space and time exactly like any other inertial observer. Minkowski had realized that many spaces (many classes of simultaneous[11] events) and many times could not exist in a three-dimensional world (i.e., if there existed just *one* space). That would be possible, he argued, only in an absolute four-dimensional world with one temporal and three spatial dimensions, where the spaces of different inertial frames can be regarded as three-dimensional "cross-sections" of it. That is why Minkowski preferred to call the relativity postulate "the *postulate of the absolute world*".

As the principle of relativity in its original formulation given first by Galileo [9] and then generalized by Poincaré [10] and Einstein [11] states that absolute uniform motion cannot be detected, Minkowski's analysis provided the answer to the difficult question that appeared to follow from that impossibility "Why does absolute uniform motion (i.e., uniform motion in the absolute space) not exist?" The answer based on Minkowski's insight turned out to be radical – absolute uniform motion does not exist because no *single* and therefore no *absolute* space exists; what exists is "an infinite number of spaces", which in turn is possible in an absolute four-dimensional spacetime where there is no motion at all (not only absolute uniform motion). What we perceive as motion of three-dimensional bodies is in fact a set of changing with time images of three-dimensional "cross-sections" of the forever given four-dimensional worldtubes of macroscopic physical bodies (since there are no three-dimensional bodies in spacetime).

The conclusion that Minkowski spacetime (sometimes also called a block universe) represents a real four-dimensional world is not merely a possible interpretation of relativity. It is the only interpretation that does not contradict the experimental evidence, which supports relativity.[12] In fact, the support from the experimental evidence is so strong that unequivocally confirms what Minkowski appeared to have realized – that *special relativity is impossible in a three-dimensional*

---

[11] It appears certain that Minkowski realized that relativity of simultaneity was impossible in a three-dimensional world and regarded it, along with length contraction, as manifestations of the four-dimensionality of the world. So, taken even alone, relativity of simultaneity is sufficient to prove the reality of spacetime.

[12] More precisely, the world at the macroscopic scale should be at least four-dimensional (with one temporal and three spatial dimensions) in order to avoid a direct contradiction with relativity. A model of a growing block universe introduced in 1923 by Broad [13] with some most recent versions [14, 16] has been regarded as an alternative to Minkowski spacetime that does not contradict relativity either. However, by explicitly assuming that the existence of physical bodies is absolute it becomes evident that the growing block universe model also contradicts relativity – the hypersurface (no matter of what shape) on which the birthing of events happens constitutes an objectively privileged hypersurface (existence is absolute!) and therefore an objectively privileged reference frame. Why existence must be regarded as absolute and cannot be relativized is briefly explained in Footnote 14.

world[13] [3]. To see why this is so, assume for a moment that it is not the case and that what exists is indeed the three-dimensional world – the present. Then, as it is the only thing that exists, all observers in relative motion should share the *same* present, i.e., the same set of simultaneous events. Therefore, simultaneity would turn out to be absolute, if the world were three-dimensional. It is then immediately seen that relativity of simultaneity does imply the reality of Minkowski space, if each of two observers in relative motion initially accepts the pre-relativistic presentist view (based on the idea of absolute simultaneity) – as the observers have different sets of simultaneous events it follows that they have different presents, i.e., different three-dimensional worlds. But this is only possible[14] if reality is a four-dimensional world, represented by Minkowski spacetime, which makes it possible for the two observers to regard two different three-dimensional "cross-sections" of it as their presents.

Another way to see why the reality of Minkowski's four-dimensional world cannot be successfully questioned is to ask explicitly whether the *experiments* which confirmed the relativistic effects would be possible if the macroscopic physical bodies involved in these experiments were three-dimensional. A definite answer to such a question directly linking experimental results with the dimensionality of macroscopic bodies and ultimately with the dimensionality of the world can settle the issue of the reality of spacetime once and for all. Analyses of length contraction, time dilation, and the twin paradox clearly demonstrate that these effects would be impossible if the macroscopic physical bodies involved in them were three-dimensional [4, 6].

---

[13] Some might object that such a claim is wrong since special relativity can be equally formulated in a three-dimensional and a four-dimensional language. They might even point out that its original formulation was in a three-dimensional language. Even if one agrees that the two representations of relativity are equivalent in a sense that they correctly describe the relativistic effects, they are entirely different in terms of the dimensionality of the world. Clearly, the world is either three-dimensional or four-dimensional. Therefore only one of the representations of relativity is correct since only one of them adequately represents the world's dimensionality at the macroscopic scale. Moreover, general relativity cannot be adequately represented in a three-dimensional language. Einstein did formulate special relativity in a three-dimensional language, but when Minkowski asked questions about the physical meaning of the relativity principle and length contraction, for example, it became clear that the relativity principle and the relativistic effects are manifestations of the four-dimensionality of the world.

[14] Strictly speaking, it appears that the three-dimensionalist view (presentism) can be preserved and made compatible with relativity of simultaneity if existence (like motion and simultaneity) is also relativized. In such a case each of two observers in relative motion will acknowledge only the existence of his or her own present and will deny the existence of the other observer's present. However, this possibility is ruled out when relativistic situations not involving relativity of simultaneity are analyzed. For instance, the twin paradox (which is an *absolute* relativistic effect) would be impossible if the twins existed only at their present moments as three-dimensional bodies as required by such a relativized version of presentism [4, 6]. Also, relativization of existence is unquestionably ruled out by taking into account (i) the existence of accelerated observers in special relativity [4] and (ii) conventionality of simultaneity [5].

Consider as an example length contraction [3]. Assume that two observers A and B in relative motion measure the same meter stick which is at rest with respect to A. As a spatially extended three-dimensional body is defined in terms of simultaneity – as "all its parts which exist *simultaneously* at a given moment of time" – relativity of simultaneity implies that the two observers measure *two* different three-dimensional objects: A and B have different sets of simultaneous events, which means that two different three-dimensional meter sticks (two different sets of simultaneously existing parts of the meter stick), one of which is shorter, exist for them. This relativistic fact – while measuring the *same* meter stick two observers in relative motion measure *two* different three-dimensional meter sticks of different lengths – reveals the deep physical meaning of length contraction first realized by Minkowski: length contraction is a manifestation of the four-dimensionality of the meter stick. The meter stick is not the three-dimensional object of our perceptions, but a four-dimensional worldtube, which consists of the ordinary three-dimensional meter stick at *all* moments of its history. The meter stick's worldtube must be a real four-dimensional object in order to allow A and B to regard two different three-dimensional "cross-sections" of it (of different lengths) as their three-dimensional meter sticks. Clearly, what is "the same meter stick" is the meter stick's worldtube.

It should be stressed that if the macroscopic physical objects and the world were three-dimensional, this effect would be *impossible* – if the meter stick's worldtube were not real, this would mean that A and B would measure the *same* three-dimensional meter stick (the same set of simultaneously existing parts of the meter stick); therefore the observers would have a common class of simultaneous events in contradiction with special relativity.

I believe even the concise arguments in this section convincingly demonstrate that Minkowski spacetime represents a real four-dimensional world in which the entire histories of all *macroscopic* three-dimensional bodies of our perceptions are realized as the bodies' worldtubes. What we perceive as interactions between three-dimensional bodies which are governed by physical laws are in reality a forever given network of worldtubes. Therefore Minkowski's program does appear to be a natural conjecture since if we can identify physical laws by studying the apparent interactions of the observed three-dimensional reflections of physical bodies, we should be able to recognize those laws by examining the network of worldtubes and to realize that the physical laws are merely interrelations between worldtubes.

Minkowski himself demonstrated how his program works in electrodynamics (in the cases of "the elementary laws formulated by A. Liénard and E. Wiechert" and "the ponderomotive action of a moving point charge on another moving point charge[15]" [1]) and in relativity by revealing the profound physical meaning of length

---

[15] Minkowski did not hide his satisfaction at the application of his program [1]:

> "When we compare this statement with previous formulations of the same elementary law of the ponderomotive action of a moving point charges on one another, we are compelled to admit that it is only in four dimensions that the relations here taken under consideration

contraction[16] – the relativistic length contraction of physical bodies and distances is a manifestation of the four-dimensionality of the bodies and the world. In the spacetime diagram depicted in Fig. 1 of his paper [1] he represented two Lorentzian electrons in relative motion by two four-dimensional bands in spacetime which form an angle between them. Each of these electrons can be regarded at rest when a separate time axis is introduce along the band of each electron. Then the spatial axis of each electron intersects the two bands in two "cross-sections" one of which represents the proper length of the electron at rest and the other – the contracted length of the moving electron. Minkowski demonstrated that the two "cross-sections" are related by the correct formula of relativistic length contraction and concluded: "But this is the meaning of Lorentz's hypothesis of the contraction of electrons in motion" [1].

It turns out that Minkowski's program is the only tool for revealing the deep physical meaning of the relativistic effects. For example, the twin paradox (as an absolute relativistic effect) was fully understood and explained only when it was realized that it is the triangle inequality in spacetime; if it is assumed that the worldtubes of the twins (which form an idealized triangle in spacetime) were not real four-dimensional objects and that the twins existed only at their present moments as the ordinary three-dimensional bodies of our perceptions, this relativistic effect would be impossible ([6], Ch. 5). By following the same approach and assuming that the worldtubes of two clocks in relative motion were not real and that the clocks existed only at their present moments as the ordinary three-dimensional clocks, it can be shown that another relativistic effect – time dilation – could not exist either ([6], Ch. 5).

> reveal their inner being in full simplicity, and that on a three dimensional space forced upon us from the very beginning they cast only a very tangled projection".

Another example of how Minkowski's program may provide a simpler and more adequate picture of some controversial issues in electrodynamics is the debate on whether or not a uniformly accelerating charge radiates (see, for example, [17], Ch. 17 and the references therein). An arbitrarily accelerated charge radiates, whereas a charge moving with constant velocity, i.e., by inertia, does not. In terms of worldlines, a charge whose worldline is curved (deformed) radiates, whereas a charge whose worldline is straight (undeformed) does not. As the worldline of an uniformly accelerating charge is also deformed the qualitative answer to the question whether or not such a charge should radiate is obviously affirmative. Then a quantitative answer is expected to be also affirmative ([17], Ch. 17 and the corresponding references therein).

[16] Minkowski specifically pointed out that length contraction is not "a consequence of resistance in the ether" [1]:

> According to Lorentz any moving body must have undergone a contraction in the direction of its motion. In particular, if the body has the velocity v, the contraction will be of the ratio
>
> $$1 : \sqrt{1 - \frac{v^2}{c^2}}\,.$$
>
> This hypothesis sounds extremely fantastical. For the contraction is not to be thought of as a consequence of resistance in the ether, but simply as a gift from above, as an accompanying circumstance of the circumstance of motion.

12.3 Newton's Three Laws and Worldlines

This section deals with Newton's three laws of motion and some old difficulties involved in our understanding of uniform and accelerated motion that are still without a satisfactory explanation. We will see that Newton's laws are in fact basic statements about worldtubes in Minkowski spacetime which provide a natural resolution of those difficulties.

The three laws of motion in Newton's original formulation are [18]:

1. Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.
2. The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
3. To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

The main difficulties in understanding inertial (uniform) and accelerated motion can be summarized as:

- Why can the state of rest and of uniform motion along a straight line (i.e., motion with constant velocity) not be experimentally distinguished? In other words, why can motion with constant velocity not be discovered?
- Why does a body moving with constant velocity appear to move on its own and can move forever if nothing prevents it from doing so?
- Why does a body resist any change in its state of motion with constant velocity? In other words, why can accelerated motion be detected?

A physical body is a forever given four-dimensional timelike worldtube in spacetime. A straight worldtube represents a body which moves with a constant velocity with respect to an inertial reference frame. We can choose the time axis of another inertial reference frame along the worldtube of this body, which means that the body will appear at rest in the new frame.[17] This provides an answer to the first question above – the worldtubes of a body at rest in an inertial reference frame and of another body moving with constant velocity with respect to the first body

---

[17] Minkowski dealt with this situation in the following way [1]:

> We now want to introduce this fundamental axiom:
>
> *The substance existing at any world-point may always, with the appropriate fixation of space and time, be looked upon as at rest.*
>
> The axiom signifies that at every world-point the expression
>
> $$c^2dt^2 - dx^2 - dy^2 - dz^2$$
>
> always has a positive value, or, what comes to the same, that any velocity v always proves less than c. Accordingly c would stand as the upper limit for all substantial velocities, and that is precisely what would reveal the deeper significance of the magnitude c.

are both straight (undeformed). The fact that all straight timelike worldtubes in spacetime are equivalent explains why the states of rest and motion with constant velocity cannot be distinguished – one can choose the time axis of an inertial reference frame along a given (arbitrarily chosen) straight timelike worldtube, which means that the body represented by that worldtube will appear at rest, whereas the bodies represented by other straight timelike worldtubes, forming an angle with the first worldtube, will appear in motion with constant velocity. Choosing the time axis along another straight timelike worldtube will provide a different description in three-dimensional language of the set of worldtubes according to which the first body will appear moving with constant velocity with respect to the new inertial frame.

When described in three-dimensional language a body, whose worldtube is straight, appears to *move on its own* with constant velocity relative to a an inertial reference frame. Since Galileo [9] (who disproved Aristotle's view that "everything that is in motion must be moved by something" [19], p. 167) it has been taken as a postulate (supported by the experimental evidence) that a body moving with constant velocity appears to move on its own which could continue forever if the body did not encounter any obstacles. This motion by inertia, which Newton postulated as his first law of motion, has never been given an explanation. The explanation became possible only after Minkowski's four-dimensional representation of special relativity, but it is completely unexpected – a body moving by inertia does not move on its own; in reality, it does not move at all since the body is a four-dimensional worldtube in the static Minkowski world. A body moving by inertia only *appears* to move on its own (and to be in motion), but it is only due to our perception reflecting the fact that the body's worldtube is straight – at each moment we perceive the light reflection from a different three-dimensional "cross-section" of the inclined straight worldtube of the body we observe and it *appears* that a three-dimensional body moves uniformly. So Newton's first law turns out to be a statement of the existence of straight timelike worldtubes in spacetime.

Whenever we talk about inertia we often mean two aspects: (i) a body moves with constant velocity *on its own* (Newton's first law), and (ii) a body *resists* any change in its state of motion with constant velocity (Newton's second law). As we saw the first aspect of inertia reflects the fact that there exist straight timelike worldtubes in spacetime. The second aspect of inertia as well as Newton's second and third laws and the last question in the list above can be best explained if we try to understand why an accelerating body resists its acceleration. For centuries all attempts to explain that resistance had been mostly seeking its origin in space, but there had been an obvious problem – if space is the cause of that resistance, then why does the motion of a body moving by inertia not be resisted by space?

Minkowski's program offers an amazing explanation of the resistance every accelerating body encounters. First, note that the worldtube of an accelerating body is *deformed* (not straight). Second, after the realization that a macroscopic physical body is a *real* four-dimensional timelike worldtube in spacetime it is natural to assume that the deformed worldtube of an accelerating body (like a deformed three-dimensional rod) *resists its deformation*. The four-dimensional stress arising in the deformed worldtube of an accelerating body gives rise to a static restoring force which tries to restore the worldtube to its initial undistorted shape. This restoring force manifests itself as the inertial force to which the accelerating body is subjected.[18] Therefore, the resistance an accelerating body encounters does not come from space but originates in the body *itself*, more precisely in the body's deformed worldtube.

Hence the answer to the question "Why does a particle resist any change in its state of motion with constant velocity?" turns out to be quite unexpected. I guess Minkowski would have been thrilled to realize that the very existence of inertia (especially its second aspect) is another manifestation of the reality of worldtubes and spacetime itself.

Now the answers to the questions of why an accelerated motion can be detected experimentally, whereas motion by inertia cannot, are also clear. Accelerated[19] motion can be detected since the worldtube of an accelerated body is deformed and resists its deformation. This four-dimensional resistance is manifested as the resistance the body offers to its acceleration. It is through this resistance any accelerated motion can be experimentally detected. The worldtube of a body moving with constant velocity is *not* deformed, which means that the body offers no resistance to its inertial motion. That is why inertial (i.e., non-resistant) motion cannot be discovered.

Newton's second law turns out to be a statement that in order to have a statically curved worldtube (or, in three-dimensional language, to accelerate a body) a force should be applied (to overcome the worldtube's static resistance to deformation). Such a force comes from another worldtube that statically curved (deformed) the first one. As the two worldtubes are mutually curved by each other in the static Minkowski spacetime, each of them resists its deformation caused by the other worldtube. Therefore, what is an external force for one worldtube (for the accelerated body) is a resistance force for the other worldtube (an inertial force acting back on the body that accelerates the first one) and vice versa. As a result of this symmetry the two forces have equal magnitudes and opposite directions, which is Newton's third law.

---

[18] Calculations of the static restoring force arising in the deformed worldtube of an accelerating body show that it does have the form of the inertial force ([6], Ch. 10).

[19] There still exists some confusion on whether acceleration is absolute in relativity. When Minkowski's program is employed it becomes clear that absolute acceleration does not imply absolute space (relative to which a body accelerates). The acceleration of a body is absolute in a sense that one does not need another body *relative* to which the first one accelerates; the acceleration of the first body can be determined in an *absolute* way by detecting the resistance the body offers to its acceleration. In special relativity any acceleration is absolute – a body whose worldtube is deformed accelerates (in three-dimensional language) in an absolute fashion. In general relativity there exist two types of acceleration: (i) absolute acceleration when a body resists its acceleration (the body's worldtube is deformed), and (ii) relative acceleration caused by the so called geodesic deviation (the bodies subjected to relative acceleration do not resist their acceleration since their worldtubes are geodesic, i.e., curved, but not deformed) ([6], Sec. 8.1).

So, Minkowski's program works perfectly in the case of Newtonian physics as well – Newton's three laws are merely statements about the existence of straight timelike worldtubes in Minkowski spacetime which, like ordinary tubes or rods, resist their *static* deformation.

12.4 Spacetime and Gravity

In this section we will see how Minkowski's program could have been consistently applied in order to arrive at the general theory of relativity. Let us start with the question: Could Minkowski have discovered general relativity by following his own program if he had lived longer? This seems unlikely if he had followed his approach to achieving new results in theoretical physics as revealed in his talk "Space and Time". There Minkowski implied that special relativity could have been discovered on the basis of mathematical considerations alone, but made it clear that that possibility was realized only after the theory was discovered [1]:

> Such a premonition would have been an extraordinary triumph for pure mathematics. Well, mathematics, though it now can display only staircase-wit, has the satisfaction of being wise after the event, and is able, thanks to its happy antecedents, with its senses sharpened by an unhampered outlook to far horizons, to grasp forthwith the far-reaching consequences of such a metamorphosis of our concept of nature.[20]

Clearly, as a mathematician Minkowski believed that pure mathematics could play a leading role in discoveries in physics. However, this view is not completely shared even by other mathematicians interested and involved in theoretical physics. Here is what Hermann Weyl wrote on this issue [20]:

> All beginnings are obscure. Inasmuch as the mathematician operates with his conceptions along strict and formal lines, he, above all, must be reminded from time to time that the origins of things lie in greater depths than those to which his methods enable him to descend. Beyond the knowledge gained from the individual sciences, there remains the task of *comprehending*. In spite of the fact that the views of philosophy sway from one system to another, we cannot dispense with it unless we are to convert knowledge into a meaningless chaos.

It is true that mathematical considerations helped Minkowski to realize that space and time are different dimensions of an absolute underlying reality – spacetime. But physical considerations played a crucial role. Minkowski himself admitted that "The views on space and time which I wish to lay before you have sprung from the soil of experimental physics" [1].

I think purely mathematical approach could not have succeeded in discovering that gravity is not a force, but a manifestation of the curvature of spacetime.

---

[20] Here we again see another indication that Minkowski does not seem to have regarded the four-dimensional world uniting space and time as a mathematical space – he talks about "such a metamorphosis of our concept of *nature*" (italics added).

Minkowski's treatment of gravity at the end of his paper "Space and Time" is not very promising since he still regarded the gravitational attraction as a force [1]:

> In mechanics as reformed in accordance with the world-postulate, the disturbing lack of harmony between Newtonian mechanics and modern electrodynamics disappears of its own accord. Before concluding I want to touch upon the attitude of Newton's law of attraction toward this postulate. I shall assume that when two mass points m, m_1 follow their worldlines, a motive force vector is exerted by m on m_1, of exactly the same form as that just given for the case of electrons, except that $+mm_1$ must now take the place of $-ee_1$.

I will now outline a possible conceptual analysis of physical facts of gravitational physics (known in 1908), which applies Minkowski's program. Such an analysis, which, as Minkowski put it, can now display only staircase-wit, demonstrates how naturally and smoothly one arrives at general relativity when the implications of Minkowski's idea of the absolute four-dimensional world are analyzed and the issue of the reality of worldtubes is taken seriously.

A conceptual analysis of Newton's gravitational theory could and should have revealed, long before Einstein realized it, that there are problems with Newton's notion of gravitational force. The first of those problems was realized by Einstein most probably in November 1907 and this insight set him on the path toward his theory of general relativity (quoted from [21]):

> I was sitting in a chair in the patent office at Bern when all of a sudden a thought occurred to me: "If a person falls freely he will not feel his own weight." I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation.

Einstein was so impressed by this insight that he called it the "happiest thought" of his life [21].

And indeed if the fall of a body in a gravitational field is conceptually analyzed it becomes clear that there is a problem with Newton's explanation that the body is falling because it is subjected to a gravitational force. According to Newton's second law $\mathbf{F} = m\mathbf{a}$ a force is necessary to accelerate a body since the body resists its acceleration (and the force should overcome that resistance). Therefore, a falling body should be subjected to a force $\mathbf{F}_g = m\mathbf{g}$, which Newton called a gravitational force, since it forces the body to accelerate with an acceleration $\mathbf{g}$.

However, the gravitational force is very different from the contact forces described by Newton's second law since it is an "action at a distance" force. Newton himself appeared to have had a lot of difficulty understanding the nature of such a non-contact force. In a letter to Richard Bentley Newton wrote [22]:

> It is inconceivable, that inanimate brute matter, should, without the mediation of something else, which is not material, operate upon and affect other matter without mutual contact, as it must be, if gravitation, in the sense of Epicurus, be essential and inherent in it. And this is one reason why I desired you would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of any thing else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers."

What is also strange with this force is what Galileo discovered through conceptual analysis[21] (and possibly through experiment[22] as well) of Aristotle's claim that heavy bodies fall faster than light ones. Galileo concluded that all bodies, no matter heavy or light, fall at the same rate, or in Newton's terms, with the *same* acceleration. This experimental fact is so bizarre that it needed another Galileo – another master of thorough conceptual analyses – to reveal what that fact has been trying to tell us. It is unclear whether Einstein, whose thought experiments made him a second Galileo in this respect, had extracted some valuable information from that fact.

A thorough analysis of Galileo's discovery could have revealed an interesting similarity between that discovery and inertial motion. According to Newton's first law of motion (and Galileo's own experiments [9] which had led him to the idea of inertial motion) different bodies move with the *same* velocity *by inertia* no matter whether they are heavy or light. So if heavy and light bodies fall with the same acceleration it is tempting the say that they move by inertia[23] and because of this it does not matter whether they are heavy or light. However, the problem is obvious – how could they move by inertia if they accelerate?

But if one does not give up and continues to analyze Galileo's strange discovery and even performs other simple experiments an insight will almost certainly enlighten such a person. When we allow drops of water to fall we can see that they are not deformed which demonstrates that the drops do not resist their downward

---

[21] Through Salviati Galileo demonstrated to Simplicio that the assumption – a heavy body falls faster than a light body – leads to a contradiction ([23], p. 446):

> If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter... But if this is true, and if a large stone moves with a speed of, say, eight while a smaller moves with a speed of four, then when they are united, the system will move with a speed less than eight; but the two stones when tied together make a stone larger than that which before moved with a speed of eight. Hence the heavier body moves with less speed than the lighter; an effect which is contrary to your supposition. Thus you see how, from your assumption that the heavier body moves more rapidly than the lighter one, I infer that the heavier body moves more slowly.

[22] Again through Salviati Galileo implies that he did perform experiments with heavy and light bodies ([23], pp. 447–448):

> Aristotle says that "an iron ball of one hundred pounds falling from a height of one hundred cubits reaches the ground before a one-pound ball has fallen a single cubit." I say that they arrive at the same time. You find, *on making the experiment* [italics added], that the larger outstrips the smaller by two finger-breadths, that is, when the larger has reached the ground, the other is short of it by two finger-breadths; now you would not hide behind these two fingers the ninety-nine cubits of Aristotle, nor would you mention my small error and at the same time pass over in silence his very large one.

[23] Asking the question of whether a falling body moves non-resistantly, on its own, does not appear to be intellectually unachievable even because Aristotle had already regarded the motion of falling bodies natural or not forced ([19], pp. 136, 199).

acceleration.[24] We can test that amazing observation by jumping from a hight, for example, and will also observe that our bodies do not resist our fall. Also, if a balloon filled with water is attached to a string, so it is prevented from falling, its shape deforms, which demonstrates that the balloon resists its state of rest in the Earth's gravitational field. However, if the balloon is allowed to fall, its shape becomes spherical, which is a clear indication that the falling balloon does not resist its fall.

These observations imply that a falling body offers no resistance to its acceleration. Therefore, it is not unthinkable to imagine that one can arrive through analyzing gravitational phenomena (not necessarily through insight like Einstein) at the conclusion that a falling body is not subjected to any gravitational force, which would otherwise be necessary if the body resisted its fall. This would mean that the falling body moves non-resistantly, by inertia. But again – how could that be since it accelerates?

Taking seriously the existence of worldtubes and following Minkowski's program makes it possible to decode the message hidden in Galileo's discovery that both heavy and light bodies fall with the same acceleration. This message turns out to be profound – gravity is a manifestation of the curvature of spacetime.

And indeed the new view of gravity follows unavoidably (not just naturally) – the worldtube of a falling body should be curved (to reflect the fact that the body accelerates), but not deformed (to account for the fact that the body does not resist its acceleration). Such a worldtube cannot exist in the flat Minkowski spacetime since if a worldtube is curved there it is also deformed. Only in curved spacetime the worldtubes of bodies moving by inertia (non-resistantly), called geodesics, are *curved but not deformed*. So, heavy and light bodies fall with the same acceleration since they indeed move by inertia; their acceleration is a manifestation of the curvature of spacetime since their worldtubes are both curved and not deformed.

In fact, a second problem with the Newtonian notion of gravitational force could have been realized before the advent of general relativity, if this notion had been rigorously examined. According to Newton's gravitational theory a body supported in a gravitational field (say, placed on a table) is subjected to the gravitational force $\mathbf{F}_g = m\mathbf{g}$, where $\mathbf{g}$ is the acceleration due to gravity. The obvious fact here is that the body does not accelerate and does not even move. But how could then a force act on the body if it does not accelerate?

---

[24] Galileo (through Salviati) virtually arrived at the conclusion that a falling body does not resist its fall ([23], p. 447):

> But if you tie the hemp to the stone and allow them to fall freely from some height, do you believe that the hemp will press down upon the stone and thus accelerate its motion or do you think the motion will be retarded by a partial upward pressure? One always feels the pressure upon his shoulders when he prevents the motion of a load resting upon him; but if one descends just as rapidly as the load would fall how can it gravitate or press upon him? Do you not see that this would be the same as trying to strike a man with a lance when he is running away from you with a speed which is equal to, or even greater, than that with which you are following him? You must therefore conclude that, during free and natural fall, the small stone does not press upon the larger and consequently does not increase its weight as it does when at rest.

It is ironic that even today this question still can confuse some physics students. One may hear "explanations" such as this one – the body does not accelerate (and does not move) since there are two exactly counterbalancing forces at work: gravity pulling the body down and the table pushing it up. So the net force acting on the body is zero, which means that the body should not accelerate (and not move).

The error here, I think, is obvious – this "explanation" answers a wrong question: "Why does the body not accelerate and not move?" The question we asked is: "Why is there a gravitational force $\mathbf{F}_g = m\mathbf{g}$ (that is balanced by the normal reaction force coming from the table), if the body does not accelerate?" The question is about the very existence (origin) of the gravitational force (that causes the normal reaction force).

Had the two problems with the notion of gravitational force – that the Newtonian gravitational theory does not have an answer to the above question and that a falling body does not offer any resistance to its acceleration – been realized before 1908 and Minkowski's program had been taken seriously, the discovery of general relativity could have been a natural and unavoidable discovery. Now many physicists feel that general relativity appeared so quickly after special relativity (which made Einstein ask what is the speed of propagation of gravity) mostly due to the lucky fact that we had Einstein.

Now general relativity regards gravity as a manifestation of the curvature of spacetime[25] and provides a consistent no-force explanation of gravitational interaction of bodies which follow geodesic paths, i.e., which are represented by geodesic worldtubes. But it is silent on the nature of the very force that has been regarded as gravitational – the force acting upon a body at rest in a gravitational field.

However, when the reality of worldtubes is taken into account and Minkowski's program is employed the picture provided by general relativity becomes complete and fully consistent. The worldtube of a body falling in a gravitational field is geodesic and the body does not resist its fall since its worldtube is not deformed.[26]

---

[25] This will be the only place in the paper where the debate over the ontological status of spacetime itself (i.e., the debate substantivalism versus relationalism) will be briefly mentioned. Spacetime must exist in order to explain gravity – if spacetime were a non-entity, no matter how glorious, it could not curve; what does not exist does not possess real properties such as curvature, which manifest itself in the real gravitational interaction.

[26] If we consider a body falling toward the surface of the Earth, according to general relativity it is the surface that accelerates since its worldtube is deformed (not geodesic), whereas the falling body moves by inertia (non-resistantly) and its worldtube is geodesic (not deformed). One can see from here why the equivalence principle works. If a body falls toward the floor of an accelerating rocket, it is the floor that accelerates (its worldtube is deformed), whereas the falling body moves by inertia (its worldtube is geodesic). However, if a body falls toward the center of the Earth, both the worldtube of the body and the worldline of the Earth's center are geodesic and no true acceleration (deformation of a worldline) is involved. The apparent acceleration between the body and the Earth's center is caused by the fact that there are no straight and no parallel worldlines in curved spacetime. The body and the Earth's center only appear to accelerate relative to each other; the rate of change of the distance between them is given by the equation of geodesic deviation [24].

But when the body is prevented from falling its worldtube is deviated from its geodesic shape and therefore deformed.

The restoring force that arises in the body's deformed worldtube turns out to be inertial since it has the same origin as in the case of an accelerating body. But in this case it is traditionally called the gravitational force. Therefore regarding the worldtubes of physical bodies as real four-dimensional objects demonstrates that the force acting on a body supported in a gravitational field is indeed *inertial* [25], which naturally explains why "there is no such thing as the force of gravity" in general relativity [26] and why inertial and gravitational forces (and masses) are equivalent ([6], Ch. 10).

12.5 Spacetime and Quantum Physics

At first sight it appears that Minkowski's program cannot be applied in quantum physics for two reasons. First, we do not know what the quantum object (e.g., an electron or a photon) is, and according to the standard interpretation of quantum mechanics we cannot say or even ask anything about the quantum object between measurements. In this sense, I think, Einstein was right that quantum mechanics is essentially incomplete. However, it is unrealistic to assume that an electron, for example, does not exist between measurements. But if it exists, it is something and we should know what that something is.

Second, although it is not clear what an electron is, it is certain what the electron is *not* – it is not a worldline in spacetime. Then, how could Minkowski's program – physical laws might find their most perfect expression as interrelations between worldlines – be applied? Here we will go beyond this program and ask whether physical laws can be expressed in terms of some spacetime structures, not worldlines.

Two things appear unquestionable: (i) relativity does not fully apply at the quantum level since its equations of motion do not describe adequately the behaviour of quantum objects, and (ii) spacetime has the same status in the quantum world as in the macroscopic world – it is the underlying reality at both levels. Therefore quantum physics should provide a spacetime model of the electron and of all quantum objects.

In an attempt to get an insight into what the spacetime model of an electron might be, let us first see why an electron is not a worldline. This, for example, can be demonstrated in the case of interference experiments performed with single electrons [27]. In such double-slit experiments accumulation of successive single electron hits on the screen builds up the interference pattern that demonstrates the wave behaviour of *single* electrons. If we look at the screen, we see that every single electron is detected as a localized entity and we are tempted to think that the electron was such an entity before it hit the screen. Our intuition leads us to assume that if the electron hits the screen as a localized entity, it is such an entity at *every* moment of time, which means that the electron exists *continuously* in time as a localized entity. But if this were the case, every single electron would behave as an ordinary particle and should go only through one slit and no interference pattern would be observed on the screen. Therefore, the inescapable conclusion is that *the electron is not a localized entity at all moments of time*, i.e., it is not a worldline in spacetime. So then, what is the electron in spacetime?

The apparent paradox – every single electron must go through both slits (in order to hit the screen where the "bright" fringes of the interference pattern form) but is always detected on the screen as a localized entity – is obviously trying to tell us something about what the electron is. How can this message be decoded? Let us start with the unquestionable facts. Every time an electron is detected it is localized *in space*. The other fact is that an electron, when not measured, is not a worldline. That is, it is not localized in space at all moments of time. What are the alternatives then?

Our intuition might suggest an obvious alternative – when not measured an electron is some kind of a fluid and for this reason it is not localized in space. However, the difficulties with this model are enormous. It is sufficient to mention just two. First, it is unexplainable that an arbitrary fraction of this fluid, i.e. a fraction of the electron charge, has never been measured. Second, when measured the electron fluid must instantaneously collapse into the small location where the electron is detected, which leads to a contradiction with relativity since a physical fluid must move at infinite speed (and also, what is infinite in one frame of reference is not infinite in another).

Is there any other interpretation of the fact that an electron, when not observed, is not localized in space at all moments of time? Or, in other words, how can the fact that an unobserved electron is not a worldline be interpreted? When it is explicitly taken into account that a worldline represents an object that *continuously* exists in time, a possible interpretation becomes almost obvious – if an electron cannot be represented by a worldline, it may mean that it does not exist continuously at all moments of time. This interpretation provides an amazingly symmetric spacetime model of the electron – no matter whether or not an electron is observed, it is always localized both in space and time or, more precisely, it is localized in spacetime. But since it exists discontinuously in time (only at some, not at *all* moments of time) it is not a point-like entity localized just in a single spacetime point.

Perhaps the best way to envisage an electron which does not exist continuously in time is to imagine that its worldline is disintegrated into its constituent four-dimensional points. Such a spacetime model of the electron represents it not as a worldline, but as a class of point-like entities (with non-zero dimensions) scattered all over the spacetime region where the electron wavefunction is different from zero.

In our three-dimensional language such an electron will appear and disappear at a given point in space and appear and disappear at a distant location in space and so on. This does not imply motion faster than light since the electron does not move as an entity which continuously exists in time from one to the another space point. Also, such an electron possesses an internal frequency of appearance and disappearance which could explain the physical meaning of the Compton frequency of the electron. In terms of the spacetime model, the Compton frequency implies that for one second an electron will be represented by 10^{20} point-like entities. When an electron is not measured it is *actually* everywhere in the spacetime region where its wavefunction is different from zero, because its constituents are scattered all over that region. So it becomes clear how such an electron can go through both slits in the double slit experiment. When the first four-dimensional point of an electron falls in a detector it is trapped there due to a jump of the boundary conditions and all its consecutive points also appear in the detector, which means that an electron is always measured as a localized entity.

An important feature of this spacetime model of the quantum object is that the probabilistic behaviour of quantum objects does not contradict at all the relativistic forever given spacetime picture of the world – the *probabilistic distribution* of the four-dimensional points of an electron in the spacetime region where the electron wavefunction is different from zero is *forever given* in spacetime.

The attempt to extend Minkowski's program to the quantum world leads to a spacetime model of the quantum object which allows to view the quantum laws governing the probabilistic behaviour of quantum objects as reflecting the spacetime probabilistic distributions of the constituents of each quantum object.

In these desperate times in quantum physics it is worth searching for a spacetime model of the quantum object, which might provide answers to the difficult and controversial questions in quantum mechanics. In this section we briefly demonstrated how such a model, based on the idea [28] that the quantum objects do not exist continuously in time, can provide a completely different and paradox-free view of quantum phenomena.[27]

12.6 Conclusions

A hundred years after Minkowski presented his paper "Space and Time" we still owe him answers to some deep questions and ideas he outlined in his paper. One of those ideas was that physical laws might find their most perfect expression as interrelations between the worldtubes of physical bodies. He himself demonstrated how this program worked in the case of several examples one of which dealt with the physical meaning of length contraction – Minkowski anticipated that this effect is a manifestation of the reality of the worldbands representing two Lorentzian electrons subjected to reciprocal length contraction.

The purpose of this paper was to examine further whether Minkowski's program can be applied to different areas of physics. It was indeed demonstrated that, when the reality of worldtubes is taken into account, not only can physical laws be regarded as interrelations between worldtubes of macroscopic bodies,

---

[27] For a more detailed conceptual account of the idea that quantum objects may exist discontinuously in time see ([6] Ch. 6).

but a deeper understanding of the corresponding phenomena can be achieved. It was shown in Section 12.3 that the mechanical pre-relativistic and pre-quantum physical laws – Newton's three laws – can be explained as statements about the existence of straight worldtubes in flat spacetime and interrelations between them; as a bonus Minkowski's program shed some light on the possible origin of inertia. In Section 12.4 the internal logic of the concept of gravity as a force was explored and it was demonstrated that Minkowski's program inescapably leads to the concept of gravity as curvature of spacetime according to which gravitational interaction is expressed as relations between geodesic worldtubes of macroscopic bodies in curved spacetime. Finally, Section 12.5 attempted to expand Minkowski's program to quantum physics and it was shown that it has the potential to shed light even on what the quantum object might be.

References

1. Minkowski, H.: Space and Time. In this volume
2. Aristotle: On the Heavens, Book I, Ch. 1. In: Adler, M.J. (ed.) Great books of the Western world, Vol. 7, pp. 357–405. Encyclopedia Britannica, Chicago (1993)
3. Petkov, V.: On the reality of Minkowski space. Found. Phys. **37** (10), 1499–1502 (2007)
4. Petkov, V.: Relativity, dimensionality, and existence. In: Petkov, V. (ed.) Relativity and the dimensionality of the world. Springer, Berlin, Heidelberg, New York, pp. 115–135 (2007)
5. Petkov, V.: Conventionality of simultaneity and reality. In: Dieks, D. (ed.) The ontology of spacetime. Philos. Found. Phys. Series **4**, 174–185. Elsevier, Amsterdam (2006)
6. Petkov, V.: Relativity and the Nature of Spacetime. Springer, Berlin Heidelberg New York (2005)
7. Petkov, V.: Is there an alternative to the block universe view? In: Dieks, D. (ed.) The Ontology of Spacetime. Philos. Found. Phys. Series, **1**, 207–228. Elsevier, Amsterdam (2006)
8. Lorentz, H.A.: The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat, 2nd edn, p. 321. Dover, Mineola, New York (2003)
9. Galileo, G.: Dialogue Concerning the Two Chief World Systems – Ptolemaic and Copernican, 2nd edn, Chap. 2 (The Second Day). University of California Press, Berkeley (1967)
10. Poincaré, H.: Sur la dynamique de l'électron. Comptes Rendues **140**, (1905) p. 1504
11. Einstein, A.: On the electrodynamics of moving bodies. In: Lorentz, H.A., Einstein, A., Minkowski, H., Weyl, H. (eds.) The principle of relativity: A collection of original memoirs on the special and general theory of relativity, pp. 37–65. Dover, New York (1952)
12. Lorentz, H.A., Einstein, A., Minkowski, H., Weyl, H.: The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity. pp 75–91. Dover, New York (1952)
13. Broad, C.D.: Scientific Thought Routledge and Kegan Paul, London (1923). In: van Inwagen, P., Zimmerman, D. (eds.) Broads defense of the growing block model is reprinted as Ch. 8. in *Metaphysics: The Big Questions*. Blackwell, Malden (1998)
14. Ellis, G.F.R.: Physics in the real universe: Time and spacetime. Gen. Rel. Grav. **38** (2006) 1797–1824, Arxiv:gr-qc/0605049
15. Christian, J.: Absolute being versus relative becoming. In: Petkov, V. (ed.) Relativity and the dimensionality of the World. Springer, Berlin (2007), Arxiv:gr-qc/0610049
16. Sorkin, R.D.: Relativity theory does not imply that the future already exists. In: Petkov, V. (ed.). Relativity and the dimensionality of the World. Springer, Berlin (2007), Arxiv:gr-qc/0703098
17. Lyle, S.N.: Uniformly Accelerating Charged Particles. Springer, Berlin, Heidelberg, New York (2008)

18. Newton, I.: The principia: mathematical principles of natural philosophy. In: Hawking, S. (ed.) On the shoulders of giants, pp. 733–734. Running Press, Philadelphia (2002)
19. Aristotle, Physics Oxford University Press, New York (2008)
20. Weyl, H.: Space-Time-Matter, p. 10. Dover, New York (1952)
21. Pais, A.: Subtle is the Lord: The Science and the Life of Albert Einstein, p. 179. Oxford, Oxford University Press (2005)
22. Bentley, R.: "Letter of Newton to Bentley, Trinity College, Jan. 17, 1692-3." In: *Works of Richard Bentley*, vol. 3 (London 1838) pp. 210–211. Quoted in: Sir Isaac Newton's Mathematical Principles of Natural Philosophy and His System of the World, p. 634. Kessinger Publishing (2003)
23. Galileo Dialogues concerning two sciences. In: Hawking, S. (ed.) On the shoulders of giants, pp. 399–626. Running Press, Philadelphia (2002)
24. Ohanian, H., Ruffini, R.: Gravitation and Spacetime, 2nd edn, p. 343. W.W. Norton, New York, London (1994)
25. Rindler, W.: Essential relativity, 2nd edn, p. 244. Springer, Berlin (1977)
26. Synge, J.L.: Relativity: The General Theory, p. 109. Nord-Holand, Amsterdam (1960)
27. Tonomura, A., Endo, J., Matsuda, T., Kawasaki, T., Exawa, H.: Demonstration of single-electron buildup of an interference pattern. Am. J. Phy., **57**, 117–120 (1989)
28. Anastassov, A.H.: Self-contained phase-space formulation of quantum mechanics as statistics of virtual particles "Annuaire de l'Universite de Sofia St. Kliment Ohridski", Faculte de Physique **81**, 135–163 (1989)